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LAMINAR PIPE FLOW IN A TRANSVERSE MAGNETIC FIELD WITH HEAT TRANSFER

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NOMENCLATURE

a,	radius of pipe;
A,	axial temperature gradient, $= \dfrac{2q}{\rho u_m C_p}$;
$A_n(\alpha)$,	function defined in equation (2);
B_0,	uniform magnetic field;
C_p,	specific heat;
d,	diameter of pipe, $= 2a$;
h,	heat-transfer coefficient;
$I_n(\alpha)$,	modified Bessel function of order n;
k,	thermal conductivity;
K,	non-dimensional pressure gradient, $= (\partial p/\partial x\, a^2)/u_m \mu$;
M,	Hartmann number, $= B_0 a(\sigma/\mu)^{\frac{1}{2}}$;
Nu,	Nusselt number, $= hd/k$;
p,	pressure;
Pe,	Péclet number, $= (\rho u_m a C_p)/k$;
q,	uniform wall heat flux;
r,	radial coordinate, non-dimensionalized by pipe radius;
T,	temperature;
T_m,	bulk mixing-cup temperature;
T_w,	wall temperature;
$u(r, \theta)$,	axial velocity, non-dimensionalized by u_m;

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u_m, mean axial velocity;
x, axial coordinate, non-dimensionalized by pipe radius.

Greek symbols
α, $M/2$;
ϵ_n, constant defined in equation (2);
θ, angular coordinate measured from direction of magnetic field;
μ, viscosity;
ρ, density;
σ, electrical conductivity.

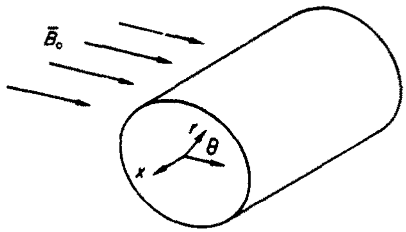

FIG. 1. Coordinate system.

INTRODUCTION

A GREAT deal of interest in Magneto-Fluid-Mechanics (MFM) has been generated in the past few years. The stimulus for much of this work has been the desire for an understanding of the influence of magnetic fields on heat and momentum transfer, with their eventual control by MFM techniques. Problems vary from the heat transfer in energy conversion devices to the aerodynamic heating and flight control of re-entry vehicles. More exotic is the possibility of explaining astrophysical phenomena such as sunspots or solar flares with MFM models.

Thus far laminar flow has been investigated more extensively than turbulent flow because of its less complicated structure. The reader is referred to Regirer's review paper [1] which presents a composite view of recent laminar MFM research along with a number of references, 130 of which were published after 1960.

The problem of laminar forced convection MFM heat transfer in transverse magnetic fields has been investigated for both flat plate and channel flows with either constant temperature or uniform heat flux conditions at the boundary [2–5]. Mittal [6] analyzed the MFM heat transfer in a circular pipe with the wall kept at a constant temperature gradient in the direction of the flow. This condition, however, causes the temperature profile and resulting heat flux at the wall to acquire an angular dependence due to the preferential symmetry of the transverse field. The present paper considers the problem of laminar MFM pipe flow in a transverse magnetic field with a prescribed uniform radial heat flux at the wall.

STATEMENT OF THE PROBLEM

An incompressible, viscous, electrically conducting fluid with constant physical properties flows through an electrically insulated circular pipe in a transverse magnetic field as shown in Fig. 1. With the assumption that viscous and joulean dissipation are negligible, the energy equation reduces to the following non-dimensional form:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial x^2} = Pe\frac{\partial T}{\partial x}U(r,\theta) \qquad (1)$$

where the axial and radial coordinates were non-dimensionalized by the pipe radius a, and the velocity by the mean velocity. Pe is the Péclet number.

The non-dimensional velocity, $u(r, \theta)$, for pipe flow in a transverse field was found in 1961 by solving the coupled electromagnetic and fluid mechanic equations [7].

$$u(r,\theta) = -\frac{K}{4\alpha}\sum_{n=0}^{\infty} A_n(\alpha)\,\epsilon_n I_n(\alpha r)\cos(n\theta) \times \{\exp[-\alpha r\cos(\theta)] + (-1)^n \exp[\alpha r\cos(\theta)]\} \qquad (2)$$

where

$$A_n(\alpha) = \left[\frac{d}{d\alpha}I_n(\alpha)\right]\Big/ I_n(\alpha)$$

and

$$\epsilon_0 = 1, \quad \epsilon_n = 2, \quad n > 0.$$

Substituting equation (2) into equation (1), one finds the equation for the temperature distribution in the flow,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial x^2} = -Pe\frac{\partial T}{\partial x}\frac{K}{4\alpha}\sum_{n=0}^{\infty} A_n(\alpha)\,\epsilon_n I_n(\alpha r)\cos(n\theta) \times \{\exp[-\alpha r\cos(\theta)] + (-1)^n\exp[\alpha r\cos(\theta)]\}. \qquad (3)$$

The uniform heat flux boundary condition in equation (3) is expressed as

$$\left.\frac{\partial T}{\partial r}\right|_{r=1} = \frac{aq}{k}. \qquad (4)$$

EXACT SOLUTION

For the case of fully developed pipe flow with a uniform heat flux at the wall, the temperature distribution can be represented as

$$T(r,\theta,x) = Ax + g(r,\theta) \qquad (5)$$

where A, the temperature gradient in the axial direction, is found by equating the heat input to the heat gained by the fluid.

Let the function $g(r, \theta)$ be represented by the form

$$g(r, \theta) = \sum_{m=0}^{\infty} f_m(r) \cos(m\theta). \tag{6}$$

Substituting equations (6) and (5) into equation (3), multiplying both sides of equation (3) by $\cos(n\theta)$ and then integrating from 0 to π, the problem becomes one of solving the following sets of equations with their respective boundary conditions.

$$m \text{ even}, \; m > 0 \quad \frac{d^2 f_0(r)}{dr^2} + \frac{1}{r}\frac{d f_m(r)}{dr} - \frac{m^2}{r^2} f_m(r) = -\frac{Pe\,AK}{\alpha}$$

$$\times \left\{A_0(\alpha) I_0(\alpha r) I_m(\alpha r) + \sum_{n=1}^{\infty} A_n(\alpha)(-1)^n I_n(\alpha r) \times [I_{n+m}(\alpha r) + I_{m-n}(\alpha r)]\right\} \tag{9}$$

$$\text{Boundary conditions,} \quad m = 0, \left.\frac{\partial f_0}{\partial r}\right|_{r=1} = \frac{aq}{k} \quad m > 0, \left.\frac{\partial f_m}{\partial r}\right|_{r=1} = 0. \tag{10}$$

Using standard mathematical techniques the solution is found to be:

$$T(r,\theta,x) = Ax - \frac{Pe\,AK}{8\alpha}\sum_{n=0}^{\infty} A_n(\alpha)(-1)^n \epsilon_n r^2 \Bigg[I_n^2(\alpha r) - I_{n+1}^2(\alpha r) - I_{n+1}(\alpha r) I_{n-1}(\alpha r) + I_n(\alpha r) I_{n+2}(\alpha r)$$

$$- \sum_{s=0}^{\infty} \frac{(\alpha r/2)^{2n+2s}(2n+2s+1)!\,n}{(2n+s+1)!\,(n+s+1)!\,(n+s)!\,s!\,(n+s+1)}\Bigg] - \sum_{\substack{m=2\\ \text{even}}}^{\infty} \cos m\theta \frac{Pe\,AK}{4m\alpha}\Bigg\{\frac{A_0(\alpha)}{m-1}[I_1(\alpha r) I_{m-1}(\alpha r)$$

$$- I_0(\alpha r) I_m(\alpha r)] r^2 + \sum_{n=1}^{\infty} \frac{A_n(\alpha)(-1)^n}{m-1} r^2 [I_{n+1}(\alpha r) I_{m+n-1}(\alpha r) - I_n(\alpha r) I_{m+n}(\alpha r) + I_{n-1}(\alpha r) I_{m-n-1}(\alpha r) - I_n(\alpha r) I_{m-n}(\alpha r)]$$

$$- \frac{A_0(\alpha)}{m+1}[I_0(\alpha r) I_m(\alpha r) - I_1(\alpha r) I_{m+1}(\alpha r)] r^2 - \sum_{n=1}^{\infty} \frac{A_n(\alpha)(-1)^n}{m+1}[I_n(\alpha r) I_{n+m}(\alpha r) - I_{n-1}(\alpha r) I_{m+n+1}(\alpha r) + I_n(\alpha r) I_{m-n}(\alpha r)$$

$$- I_{n+1}(\alpha r) I_{m-n+1}(\alpha r)] - \frac{A_0(\alpha)}{m-1}[I_1(\alpha) - I_{m-1}(\alpha) - I_0(\alpha) I_m(\alpha)] r^m - \sum_{n=1}^{\infty} \frac{A_n(\alpha)(-1)^n}{m-1} r^m [I_{n+1}(\alpha) I_{m+n-1}(\alpha)$$

$$- I_n(\alpha) I_{m+n}(\alpha) + I_{n-1}(\alpha) I_{m-n+1}(\alpha) - I_n(\alpha) I_{m-n}(\alpha)] - \frac{A_0(\alpha)}{m+1}[I_0(\alpha) I_m(\alpha) - I_1(\alpha) I_{m+1}(\alpha)] r^m$$

$$- \sum_{n=1}^{\infty} \frac{A_n(\alpha)(-1)^n}{m+1} r^m [I_n(\alpha) I_{n+m}(\alpha) - I_{n-1}(\alpha) I_{n+m+1}(\alpha) + I_n(\alpha) I_{m-n}(\alpha) - I_{n+1}(\alpha) I_{m-n+1}(\alpha)]\Bigg\}. \tag{11}$$

$$m \text{ odd}, \quad \frac{d^2 f_m(r)}{dr^2} + \frac{1}{r}\frac{d f_m(r)}{dr} - \frac{m^2}{r^2} f_m(r) = 0 \tag{7}$$

$$m = 0, \quad \frac{d^2 f_0(r)}{dr^2} + \frac{1}{r}\frac{d f_0(r)}{dr} = - \frac{Pe\,AK}{2\alpha}\sum_{n=0}^{\infty} A_n(\alpha)\,\epsilon_n(-1)^n I_n(\alpha r) \tag{8}$$

APPROXIMATE SOLUTION

The complex nature of the exact solution equation (11), makes numerical calculation of the temperature distribution and Nusselt number rather difficult. Instead, an approximation of the velocity profile equation (2) is made to give the needed simplification.

A first attempt was made to expand equation (2) for small Hartmann numbers to terms of order $(M/2)^4$. Although

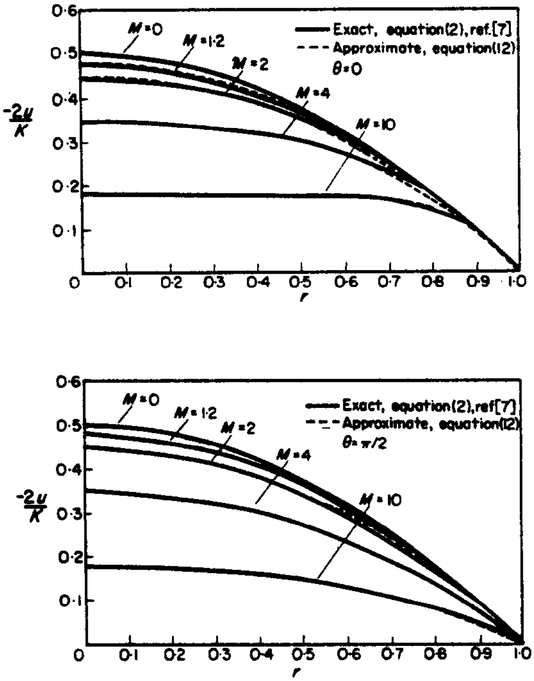

FIG. 2. Comparison of equation (12) with previously published computation of the exact velocity profile· equation (2), at $\theta = 0$ and $\theta = \pi/2$. Curves coincide except where indicated. Similar agreement is found at intermediate values of θ.

some forty-six terms were retained, the expansion deviated considerably from equation (2) in the wall region for all but very small Hartmann numbers. Keeping higher order terms did not significantly correct this problem and the method was abandoned in favor of the following scheme.

A simple function, equation (12), was constructed to produce the velocity profiles. With guidance from the low Hartmann numbers to terms of order $(M/2)^4$. Although way that it gives the correct centerline velocity and has a zero value at the wall. Three constants are used to control the flattening as the Hartmann number increases. Equation (12) is compared to Gold's computation of equation (2) in Fig. 2.

$$u(r, \theta) = -\frac{KA_0(\alpha)}{2\alpha}[(1 - ar^2 - br^4) + (ar^2 + br^4 - r^\beta)\cos^2\theta] \qquad (12)$$

where

$$A_0(\alpha) = \left[\frac{\mathrm{d}}{\mathrm{d}\alpha} I_0(\alpha)\right] \Big/ I_0(\alpha)$$

$$a = \frac{14}{14 + 1{\cdot}43\,\alpha}$$

$$b = \frac{1{\cdot}43\,\alpha}{14 + 1{\cdot}43\,\alpha}$$

$$\beta = 2{\cdot}0 + 0{\cdot}175\,\alpha^{2{\cdot}1}.$$

Substituting equation (12) into equation (1) and applying the boundary condition equation (4) to the solution, the temperature profile is found to be

$$T(r, \theta, x) = Ax - \frac{Pe\,AK}{2\alpha} A_0(\alpha) \times \left\{\left(\frac{r^2}{4} - \frac{ar^4}{32} - \frac{br^6}{72} - \frac{r^{\beta+2}}{2(\beta+2)^2}\right) + \left[\left(-\frac{a}{12} - \frac{3b}{64} + \frac{1}{8\beta} + \frac{1}{8(\beta+4)}\right)r^2 + \frac{a}{24}r^4 + \frac{b}{64}r^6 - \frac{r^{\beta+2}}{8\beta} + \frac{r^{\beta+2}}{8(\beta+4)}\right]\cos 2\theta\right\}. \qquad (13)$$

Equation (12) is seen to be a good representation for the velocity for Hartmann numbers up to ten in Fig. 2, but at high Hartmann numbers equation (12) underestimates the amount of flattening since only a fourth order polynomial was used.

For large Hartmann numbers it would be more appropriate to use the high Hartmann number approximation of equation (2) [7].

$$u(r, \theta) = -\frac{K}{2\alpha}\left[(1 - r^2\sin^2\theta)^{\frac{1}{2}} - \frac{1}{2\alpha(1 - r^2\sin^2\theta)}\right] \qquad (14)$$

$\alpha \gg 1$.

Again substituting equation (14) into equation (1) and applying the boundary condition equation (4) to the solution, the temperature profile is found to be

$$T(r, \theta, x) = Ax\frac{Pe\,AK}{2\alpha}\left[\sum_{n=0}^{\infty}\frac{r^{2n+2}}{(2n+2)^2}\left(\frac{C_n^2}{2n-1} + \frac{C_n}{2\alpha}\right) + \sum_{\substack{m=2,\\ \text{even}}}^{\infty}\cos(m\theta)\sum_{n=m/2}^{\infty}\frac{b_{nm} + d_{nm}}{(2n+2)^2 - m^2} \times \left(r^{2n+2} - \frac{2n+2}{m}r^m\right)\right] \qquad (15)$$

where

$$C_n = \frac{(2n)!}{2^{2n}n!\,n!}$$

and

$$b_{nm} = \frac{2C_n^2(-1)^{m/2}n!\,n!}{(2n-1)(n+m/2)!\,(n-m/2)!}$$

$$d_{nm} = \frac{C_n(-1)^{m/2}n!\,n!}{\alpha(n+m/2)!\,(n-m/2)!}.$$

The influence of the magnetic field on the heat transfer is found by calculating the Nusselt number which is defined as

$$Nu = \frac{hd}{k} = \frac{2aq}{k(T_w - T_m)}. \quad (16)$$

T_w is found by evaluating equations (13) or (15) at $r = 1$ and T_m is the bulk mixing-cup temperature defined by

$$T_m = \frac{\int_0^{2\pi}\int_0^1 u(r,\theta)\, T(r,\theta,x)\, r\, dr\, d\theta}{\int_0^{2\pi}\int_0^1 u(r,\theta)\, r\, dr\, d\theta}. \quad (17)$$

The results of using equations (12) and (13) and then equations (14) and (15) to calculate T_w and T_m are given in Fig. 3 where the local Nusselt number is presented as a function of the angular coordinate.

The average Nusselt number, which characterizes the overall heat transfer, is found by averaging the local Nusselt number of Fig. 3 and is presented in Fig. 4.

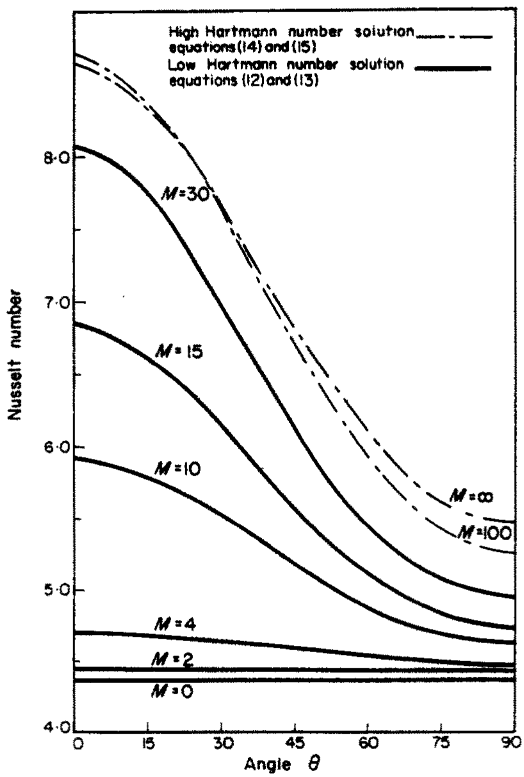

FIG. 3. Local Nusselt number as a function of the angular coordinate measured from the direction of the magnetic field.

DISCUSSION AND CONCLUSION

The exact solution, equation (11), is of such a complicated nature that it was not practical to try to calculate the bulk mixing-cup temperature, equation (17). Instead, two approximate velocity profiles were used to simplify this problem. The first function was constructed with guidance from the exact solution to produce reasonably accurate velocity profiles for low to moderate Hartmann numbers. The second was the high Hartmann number approximation of the exact profile.

Figures 3 and 4 illustrate the influence of the magnetic field on the heat transfer. In Fig. 3 it is seen that for $M < 2$ there is little angular dependence on the Nusselt number. In the limit $M \to 0$ the Nusselt number attains the value of 48/11 which is the classical value found for the case of no magnetic field. As the Hartmann number increases the Nusselt number increases, but more rapidly at $\theta = 0$ than at $\theta = \pi/2$. This is a result of the fact that the $\theta = 0$ profile suffers more flattening than the $\theta = \pi/2$ profile. Along $\theta = 0$ the velocity is decelerated in the core and accelerated in the wall region causing the flat or uniform profile which is similar to the Hartmann profile of channel flow. At $\theta = \pi/2$ the flow is decelerated only, but more strongly in the core than near the wall which results in a slight flattening effect. In the limit of $M \to \infty$, the $\theta = 0$ profile is completely flat whereas the $\theta = \pi/2$ profile is only slightly flattened and still appears parabolic [8]. Thus, the local Nusselt number, although dependent on the entire flow field, reflects the amount of flattening its local velocity profile suffers.*

The average Nusselt number in Fig. 4 shows that there is a region of interaction of about two orders of magnitude of Hartmann number over which the Nusselt number varies. For $M < 1$ there is no significant influence of the magnetic field on the heat transfer. For $1 < M < 100$† the average Nusselt number increases to the asymptotic value of about 7·0. For higher magnetic fields there is no further change in the heat transfer. It can also be seen that equation (14) overestimates the amount of flattening for low Hartmann numbers and equation (12) underestimates the flattening for high Hartmann numbers. The dashed line in Fig. 4 is a suggested interpolation between the two solutions in the intermediate region.

Although the results of this work come from the formulation of a laminar flow problem, they are related to the heat transfer in a turbulent flow since a high enough magnetic field can suppress turbulence to such a degree that the flow

* For the case of a uniform or flat velocity profile with no angular dependence the Nusselt number is 8·0. Figure 3 shows the trend towards this value at $\theta = 0$.

† These limits over which the magnetic field affects the average heat transfer are in good agreement with those found in the investigations of the MFM channel flow with a uniform heat flux [4, 5].

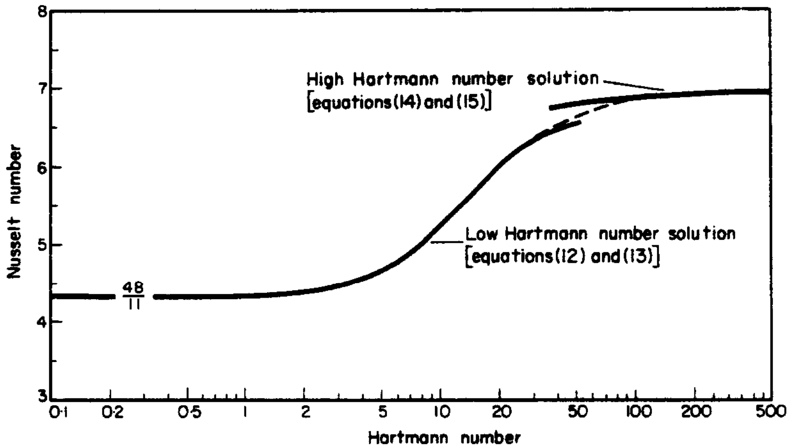

FIG. 4. Average Nusselt number as a function of the Hartmann number. Dashed line indicates suggested interpolation between the high and low Hartmann number solutions.

becomes laminar. The local and average Nusselt numbers obtained here for the case of high Hartmann numbers should also describe the heat transfer of such a flow.

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DEVICE FOR PRODUCTION OF GAS-FREE LIQUIDS OR VAPORS

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SEVERAL years ago at the University of Minnesota it was necessary to produce gas-free liquids for use in nucleate boiling experiments. Using Taylor's [1] idea of fractionation we produced a degassing unit shown in Fig. 1 which operates